PROBABILITY

Probability is a measure of how likely a particular outcome to an event is to happen.

It ranges from 0 to 1. A probability of 0 means that the outcome cannot happen. A probability of 1 means that the outcome will definitely happen. And in between 0 and 1 means that the outcome may happen.

Example with a coin

When a coin is tossed the outcome (or event) can be heads or tails. What is the probability it is tails?

Since each outcome, heads or tails, is equally likely we can say that the probability of each is 0.5.

$$P(\text{ coin toss is tails }) = \frac{1}{2}$$

Basic rule of probability

More generally we can say that where there are n equally likely outcomes then the probability of each of these possibilities will be $\frac{1}{n}$.

So we can say that

$$P(\text{outcome}) = \frac{\text{number of ways it can happen}}{\text{total number of possible outcomes}}$$

This is the basis of all probability questions in the GMAT.

Example with die

What is the probability of rolling a 6 when you throw a 6 sided die.

Each number from 1 to 6 is equally likely to be thrown and only one of those outcomes is a 6 so using the general rule we can say that

$$P(\text{ throw a six }) = \frac{1}{6}$$

Example with cards

If you pick a card at random from a deck of cars what is the probability that it is an ace?

There are 52 cards in a pack and of those there are 4 aces so

$$P(\text{ an ace }) = \frac{4}{52} = \frac{1}{13}$$

The probability two outcomes for independent events both occur can be found by multiplying their probabilities.

$$P(\text{ A and B }) = P(A) \times P(B)$$

Example with coins

What is the probability of throwing two heads in a row when tossing a coin?

This is the same as asking what the probability that the first coin tossed will be head AND the second coin tossed will be a head.

So the probability that of tossing two heads in a row is $\frac{1}{4}$.

Example with a jar

A jar contains 2 red balls and 4 green balls. What is the probability that two balls selected at random from the jar are both green?

Each ball is equally likely to be selected from the jar so we can work out the probability of the first ball selected being green.

Here is where we need to be careful, once we have taken 1 green ball out of the jar, the jar contains only 3 green balls and 2 red balls so

Now we can say that

So the probability that of picking out two green balls is $\frac{2}{5}$.

Total Probability

Events H1;H2; : : : ;Hn form a partition of the sample space S if
(i) They are mutually exclusive (Hi ∉ Hj = ;; i 6= j) and
(ii) Their union is the sample space S;
Sn
i=1 Hi = S:
The events H1; : : : ;Hn are usually called hypotheses and from their definition follows that P(H1) +
∉ ∉ ∉ + P(Hn) = 1 (= P(S)):
Let the event of interest A happens under any of the hypotheses Hi with a known (conditional) probability
P(AjHi): Assume, in addition, that the probabilities of hypotheses H1; : : : ;Hn are known. Then P(A)
can be calculated using the total probability formula.
Total Probability Formula.
P(A) = P(AjH1)P(H1) + ∉ ∉ ∉ + P(AjHn)P(Hn):
The probability of A is the weighted average of the conditional probabilities
P(AjHi) with weights
P(Hi):

Bayes Formula. Let the event of interest A happens under any of hypothesesHi with
a known (conditional) probability P(AjHi): Assume, in addition, that the probabilities
of hypotheses H1; : : : ;Hn are known (prior probabilities). Then the conditional (posterior) probability of the hypothesis Hi; i = 1; 2; : : : ; n, given that event A happened, is
P(HijA) = P(AjHi)P(Hi)/P(A) ;
where
P(A) = P(AjH1)P(H1) + …….+ P(AjHn)P(Hn):

Assume that out of N coins in a box, one has heads at both sides. Such "two-headed" coin can be

purchased in Spencer stores. Assume that a coin is selected at random from the box, and without inspecting
it, flipped k times. All k times the coin landed up heads. What is the probability that two headed coin was
selected?
Denote with Ak the event that randomly selected coin lands heads up k times. The hypotheses areH1-the
coin is two headed, and H2 the coin is fair. It is easy to see that P(H1) = 1=N and P(H2) = (N ¡ 1)=N.
The conditional probabilities are P(AkjH1) = 1 for any k, and P(AkjH2) = 1=2k:
By total probability formula,
P(Ak) =
2k + N ¡ 1/2kN

and
P(H1jAk) =2k/2k + N - 1:
Conditional Probability

Problem: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

Analysis: This problem describes a conditional probability since it asks us to find the probability that the second test was passed given that the first test was passed. In the last lesson, the notation for conditional probability was used in the statement of Multiplication Rule 2.

Multiplication Rule 2: When two events, A and B, are dependent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

The formula for the Conditional Probability of an event can be derived from Multiplication Rule 2 as follows:

Step 1: $P(A \text{ and } B) = P(A) \cdot P(B|A)$ Start with Multiplication Rule 2.

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The formula for the Conditional Probability of an event can be derived from Multiplication Rule 2 as follows:

Step 1:	$P(A \text{ and } B) = P(A) \cdot P(B\|A)$	Start with Multiplication Rule 2.
Step 2:	$\dfrac{P(A \text{ and } B)}{P(A)} = \dfrac{P(A) \cdot P(B\|A}{P(A)}$	Divide both sides of equation by P(A).
Step 3:	$\dfrac{P(A \text{ and } B)}{P(A)} = \dfrac{P(A) \cdot P(B\|A}{P(A)}$	Cancel P(A)s on right-hand side of equation.
Step 4:	$\dfrac{P(A \text{ and } B)}{P(A)} = P(B\|A)$	Commute the equation.
	$P(B\|A) = \dfrac{P(A \text{ and } B)}{P(A)}$	We have derived the formula for conditional probability.

Now we can use this formula to solve the problem at the top of the page.

Problem: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

Solution: $P(\text{Second}|\text{First}) = \dfrac{P(\text{First and Second})}{P(\text{First})} = \dfrac{0.25}{0.42} = 0.60 = 60\%$