Number Theory - Tips & Tricks

1.Sum of natural numbers from 1 to n

$$\frac{n(n+1)}{2}$$

e.g Sum of natural numbers from 1 to 40 = 40(40+1)/2 = 820

2.Sum of squares of first n natural numbers is =

$$\frac{n(n+1)(2n+1)}{6}$$

3.Sum of the squares of first n even natural numbers is

$$\frac{2}{3}n(n+1)(2n+1)$$

4.Sum of cubes of first n natural numbers is

$$\left[\frac{n(n+1)}{2}\right]^2$$

5.Any number N can be represented in the decimal system of number as

$$N = n_k 10^k + n_{k-1} 10^{k-1} + n \ldots + n_i 10 + n_0$$

Important Formulas

i. $(a + b)(a - b) = (a^2 - b^2)$

ii. $(a + b)^2 = (a^2 + b^2 + 2ab)$

iii. $(a - b)^2 = (a^2 + b^2 - 2ab)$

iv. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

v. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

vi. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

vii. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

viii.When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

xi. (a + b)^ 2 = (a^ 2 + b^ 2 + 2 ab) =(a - b)^2 + 4ab

x. (a - b)^ 2 = (a^ 2 + b^ 2 - 2 ab) = (a + b)^2 - 4ab

Some more tips:

1) k(a + b + c) = ka + kb + kc

2) (a + b) (c + d) = ac + ad + bc + bd

3) (x + a) (x + b) = x^2 + (a + b)x + ab

4) (a + b)^ 2 - (a - b)^ 2 = 4ab

5) (a + b)^ 2 - (a - b)^ 2 = 2(a^2 + b^2)

6) (a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + 3a^2b +3ab^2 + b^3

7) (a - b)^3 = a^3 - b^3 - 3ab(a - b) = a^3 - 3a^2b +3ab^2 - b^3

8) 1/a + 1/b = a + b/ ab

9) (x + a)(x + b)(x + c) = x^3 + (a +b +c)x^2 + (ab + bc + ca)x +abc

10) (a + b + c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2a +3b^2c + 3c^2a + 3c^2b + 6abc

Tests of Divisibility :

1.A number is divisible by 2 if it is an even number.

2.A number is divisible by 3 if the sum of the digits is divisible by 3.

3.A number is divisible by 4 if the number formed by the last two digits is divisible by 4.

4.A number is divisible by 5 if the units digit is either 5 or 0.

5.A number is divisible by 6 if the number is divisible by both 2 and 3.

6.A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

7.A number is divisible by 9 if the sum of the digits is divisible by 9.

8.A number is divisible by 10 if the units digit is 0.

9.A number is divisible by 11 if the difference of the sum of its digits at odd places and the sum of its digits at even places, is divisible by 11.

some more tips

1) A number is divisible by 12, when it is divisible by both 3 and 4.

2) A number is divisible by 25, when the last two digits are 00 or divisible by 25

3) A number is divisible by 125, if the last three digits are 000 or divisible by 125

4) A number is divisible by 27, if the sum of the digits of the number is divisible by 27.

5) A number is divisible by 125, if the number formed by last three digits is divisible by 125.

6) Number of the form 10^n-1 (where 'n' is a natural number) is always divisible by

7) if 'n' is even, such numbers are divisible by 11 also.

H.C.F and L.C.M :

H.C.F stands for Highest Common Factor. The other names for H.C.F are Greatest Common Divisor (G.C.D) and Greatest Common Measure (G.C.M).

The H.C.F. of two or more numbers is the greatest number that divides each one of them exactly.

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

Two numbers are said to be co-prime if their H.C.F. is 1.

Finding L.C.M and H.C.F of Fractions

$$\text{L.C.M} = \frac{\text{L.C.M of the numerators}}{\text{H.C.F of the denominators}}$$

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Product of two numbers = Product of their H.C.F. and L.C.M.