IMPORTANT FORMULA AND EQUATIONS

1. Minute Spaces:

The face or dial of watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

Hour Hand and Minute Hand:

A clock has two hands, the smaller one is called the hour hand or **short hand** while the larger one is called **minute hand** or **long hand** .

2.

i. In 60 minutes, the minute hand gains 55 minutes on the hour on the hour hand.

ii. In every hour, both the hands coincide once.

iii. The hands are in the same straight line when they are coincident or opposite to each other.

iv. When the two hands are at right angles, they are 15 minute spaces apart.

v. When the hands are in opposite directions, they are 30 minute spaces apart.

vi. Angle traced by hour hand in 12 hrs = 360°

vii. Angle traced by minute hand in 60 min. = 360°.

viii. If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast .

ix. On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow .

3. Odd Days:

We are supposed to find the day of the week on a given date.

For this, we use the concept of 'odd days'.

In a given period, the number of days more than the complete weeks are called **odd days** .

4. Leap Year:

(i). Every year divisible by 4 is a leap year, if it is not a century.

(ii). Every 4th century is a leap year and no other century is a leap year.

5. Ordinary Year:

The year which is not a leap year is called an ordinary years . An ordinary year has 365 days.

6. Counting of Odd Days:

1. 1 ordinary year = 365 days = (52 weeks + 1 day.)

1 ordinary year has 1 odd day.

2. 1 leap year = 366 days = (52 weeks + 2 days)

1 leap year has 2 odd days.

3. 100 years = 76 ordinary years + 24 leap years

= (76 x 1 + 24 x 2) odd days = 124 odd days.

= (17 weeks + days) = 5 odd days.

Number of odd days in 100 years = 5.

Number of odd days in 200 years = (5 x 2) = 3 odd days.

Number of odd days in 300 years = (5 x 3) = 1 odd day.

Number of odd days in 400 years = (5 x 4 + 1) = 0 odd day.

Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd days.

2. Day of the Week Related to Odd Days:

No. of days:	0	1	2	3	4	5	6
Day:	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

KEY NOTES:

Clocks Concept :

- The dial of the clock is circular in shape and was divided into 60 equal minute spaces
- 60 minute spaces traces an angle of 360^0
  ∴ 1minute space traverses an angle of 6^0
- In 1 hour, Minute hand traverses 60 minute space or 360^0,Hour hand traverses 5 minute space or 30^0

- The hands of the clock are perpendicular in 15 minute spaces apart

- The hands of the clock are in straight line and apposite to each other in 30 minute spaces apart.

- The hands of the clock are in straight line when they coincide or opposite to each other.
- The hands of the clock are perpendicular to each other for 22 times in 12 hours and for 44 times in a day.

- The hands of the clock are opposite to each other for 11 times in 12 hours and 22 times in a day.

- The hands of the clock coincides with each other for 11 times in 12 hours and 22 times per day

- The hands of the clock are 44 times in a straight line per day

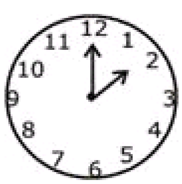
- The minute hand gain 55 minutes over hour hand per hour.

- Hence x minute space to be gained by minute hand over hour hand can be

x.(60/55) or x.(12/11)

- calculated as
- **Ex :** At what time between 2'O clock and 3'O clock the hands of the clock are opposite to each other.
  1. 34(6/11) past 2'Oclock 2. 43(7/11) past 2'Oclock
  3. 56(8/11) past 2'Oclock 4. 64(9/11past 2'Oclock

  Sol At 2'O clock the minute hand will be at 12 as shown below



The minutes hand to coincide with the hour hand it should trace at first 10 minute spaces
And then the hands of the clocks to be opposite to each other minute hand should trace 30 minute spaces i.e. totally it should gain 10+30=40 minute spaces to be opposite to that of hour hand

We know that,
Minute hand gains 55 minute spaces over hour hand in 1 hour

∴ Minute hand gain 40 minute spaces over hour hand in 40 × (60/55) = 43(7/11)
Hence the hand of the clock will minutes be opposite to each at 43(7/11) past 2'Oclock

∴ Correct option is 2'

- **When clock is too fast, too slow**
  If a clock or watch indicates 6 hr 10 min when the correct time is 6, it is said that the clock is 10 min too fast

  If it indicates 6. 40 when the correct time is 7, it is said to be
  20 min too slow.

  Now let us have an example based on this concept

  Ex My watch, which gains uniformly, is 2 min, & show at noon on Sunday, and is 4 min 48 seconds fast at 2 p.m on the following Sunday when was it correct ?

  Sol: From Sunday noon to the following Sunday at 2 p.m there are 7 days 2 hours or 170 hours.

  The watch gains $\left(2+4\frac{4}{5}\right)$ min in 170 hrs.

  ∴ the watch gains 2 min in $\left(\frac{2}{6\frac{4}{5}}\times 170\right)$ hrs i.e., 50 hours
  Now 50 hours = 2 days 2 hrs.
  ∴ 2 days 2 hours from Sunday noon = 2 p.m on Tuesday.

Calendars Concept :

- The time in which the earth travels round the sun is a solar year and is equal to 365 days 5 hrs. 48 minutes and $47\frac{1}{2}$ seconds
- Year is 365.2422 days approximately.
- The common year consists of 365 days.
- The difference between a common year and a solar year is therefore 0.2422 of a day and we consider it by adding a whole day to every fourth year.
- Consequently in every 4 th year there are 366 days.
- The years which have the extra day are called leap years. The day is inserted at the end of February, The difference between 4 common years and 4 solar years is 0.969 of a day.
- If therefore, we add a whole day to every 4 th year, we add too much by 0.0312 of a day. To take account of this, we omit the extra day three times every 400 years,
- The thing is to ensure that each season may fall at the same time of the year in all years. In course of time, without these corrections, we should have winter in July and summer in January also .

- With the very small variation, the present divisions of the year are those given in B.C 46 by Julius Caesar . The omission of the extra day three times in 400 years is called the Gregorian Correction. This correction was adopted at once in 1582 in Roman Catholic Countries. but not in England until, 1752.
  The Gregorian mode of reckoning is called the New Style, the former, the Old Style. The New Style has not yet been adopted in Russia , so that they are now 13 days behind us as an Example What we call Oct. 26 th they call 13 th Oct . They have Christmas day on 7 th of January and we have on 25 th December every year.
- In an ordinary year there are 365 days i.e., 52 weeks + 1 day
  Therefore an ordinary years contains 1 odd day.
  A leap year contains 2 odd days.

  100 year = 76 ordinary years + 24 leap years.
  = 76 odd days + 48 odd days
  = 124 odd days = 17 weeks + 5 days.(in the consideration of weeks)
  $\therefore$ 100 years contain 5 odd days.
  200 years contains 3 odd days.
  300 years contain 1 odd days
- Since there are 5 odd days in 100 years, there will be 20 days in 400 years. But every 4 th century is a leap year.
  $\therefore$ 400 years contain 21 days. Here 400 years contain no odd days.
- As First January 1 A.D was Monday. we must count days from Sunday
  i.e Sunday for 0 odd days , Monday for 1 odd day , Tuesday for 2 odd days and so on.
- Last day of a century cannot be either Tuesday. Thursday or Saturday.
- The first day of a century must either be Monday. Tuesday, Thursday or Saturday.
- Now let us observe the Examples

  Ex How many times does the 29 th days of the month occur in 400 consecutive years

  1) 97 times 2) 4400 times 3) 4497 times 4) none

  Sol: In 400 consecutive years there are 97 leap years. Hence in 400 consecutive years, February has the 29 th day 97 times, and the remaining 11 months have the 29 th day 400 x 11 or 4400 times.
  $\therefore$ 29 th day of the month occurs (4400 + 97) or 4497 times

  Ex Given that on 10 th November 1981 is Tuesday, what was the day on 10 th November 1581

  1) Monday 2) Thursday 3) Sunday 4) Tuesday

  Sol: After every 400 years, the same day comes.
  Thus if 10 th November1981 was Tuesday, before 400 years i.e on 10 th November 1581 , it has to be Tuesday.